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'Algebraic Curves with Symmetries,
their Jacobians and Integrable Dynamical Systems'
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Recent progress in the algebro-geometric construction of non-abelian monopoles

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Nonabelian gauge theories possess topologically nontrivial classical solutions that have been the focus of much attention over the years. The ADHM construction of instantons is algebraic whereas the the ADHMN construction of monopoles requires the solution of Nahm's equation, an ODE (together with suitable boundary data), and then utilising this the solution of a further Weyl equation. For an $\mathfrak{su}(2)$ gauge group some very special solutions to Nahm's equations are known but even for charge 2 the full reconstruction of the gauge fields has yet to be carried out. Whilst Nahm's equation admits algebro-geometric integration, the Weyl equation has in almost all cases only been solved numerically. We develop here ideas of Nahm to show that the both equations admit integration in terms of theta-functions utilizing Krichever's method of integration via Baker-Akhiezer functions.

Two explicit examples of non-Abelian monopoles

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In spite of many decades of intensive development of the Yang-Mills-Higgs-Nahm theory only a few explicit examples of static states — monopoles — are known for $\mathfrak{su}(2)$ and higher gauge groups. We shall illustrate the first talk with two examples for the $\mathfrak{su}(2)$ theory. The first is the algebro-geometric construction of a new particular monopole curve for charge 3 within the Hitchin approach. The second example solves in closed form the associated Weyl equation for the charge 2 monopole on the basis of the Nahm Ansatz.

SU(3) magnet and generalized theta-functions of trigonal curve

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We consider an integrable system on a coadjoint orbit of the SU(3) loop group, which we call isotropic SU(3) Landau-Lifshits equation. This system serves as a continuum limit for the SU(3) magnetic chain, as declared in the title. From the algebraic point of view the system lives on a generalized Jacobian of the trigonal curve $w^3 = w\mathcal{I}_{2N}(z) + \mathcal{I}_{3N}(z)$, and requires generalized theta-functions for a finite-gap integration. We perform separation of variables in the manner of Sklyanin. Then we integrate the system in the case of gap 1, whilst the curve is of the lowest genus.

KP hierarchy for a cyclic quiver

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We introduce a generalisation of the KP hierarchy intimately related to the cyclic quiver on m vertices, and show that it admits solutions whose poles move according to the Calogero-Moser system for the complex reflection group $S_n \wr \mathbb{Z}_m$. This is joint work with Alexey Silantyev.

KP hierarchy for a cyclic quiver

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We discuss the properties of generalized $\wp$ and σ functions associated with plane curves of genus g (here we consider $g = 2$, 3 only). These functions satisfy interesting nonlinear integrable PDEs, linear parabolic PDEs, and addition formulae. The PDEs can all be be generated from partial derivatives of a single hypersurface in each case. We show also that coefficients of the sigma expansion in genus 3 satisfy a complicated linear recurrence relations.

Algebro-geometric solutions of the Schlesinger system from conformal field theory

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It is shown that the isomonodromic tau-function can be presented as a Fourier transformation of conformal blocks. In particular, the conformal blocks of Ashkin-Teller model leads to algebro-geometric solution for the tau-function of isomononodromic deformation found by A.Kitaev and D.Korotkin.

The talk is based on joint paper: N.Iorgov, O.Lisovyy, J.Teschner, arXiv:1401:6104.

An application of the theory of tau functions to Riemann's theta function

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Using Sato's theory of KP-hierarchy we determine the top term of the series expansion of Riemann's theta function of any Riemann surface at an arbitrary point on the theta divisor. The result gives a refinement of the classical Riemann's singularity theorem.

On the factorized spectral problems and related integrable dynamical systems

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Factorized spectral problems, tensor Poisson structures and related with them integrable dynamical systems are discussed.

Elliptic and higher genus curves, classical r-matrices and integrable systems

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We will discuss integrable finite-dimensional hamiltonian systems and soliton equations possessing Lax pairs with elliptic and more complicated dependence on spectral parameter. We will construct the corresponding classical r-matrices and provide some new examples of the associated integrable systems. Among them there are generalizations of Gaudin models with and without an external magnetic field (Gaudin models generalize integrable Euler tops), different generalizations of Landau-Lifshitz and anisotropic chiral field equations etc.

Bergman tau-function and Witten cycles in moduli spaces

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The Bergman tau-function can be viewed as a generalization of Dedekind's eta-function to higher genus. In physical terms, it is a partition function of free bosons on a Riemann surface with flat singular metric. Mathematically, it arises as isomonodromic tau-function of special Riemann-Hilbert problems; it also appears in the problem of holomorphic factorization of determinant of Laplace operator. Analyzing analytical properties of the tau-function one can derive several (both new and known before) results on geometry of moduli spaces (including, for example, Mumford's relation between determinantal line bundles on moduli spaces). Applying similar machinery to moduli spaces $M_{g,n}$ of punctured Riemann surfaces, one can give an analytical derivation of the formula for Hodge class on $M_{g,n}$ in terms of Witten's cycles, which are special combinatorially defined submanifolds of $M_{g.n}$. The talk is based on joint works with M. Bertola, A. Kokotov and P. Zograf.

Integrable Dispersive Chains

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In this talk we consider integrable dispersive chains associated with the so called Energy Dependent Schrodinger operator. In a general case multi component reductions of these dispersive chains are new integrable systems, which are characterised by two arbitrary natural numbers. Also we show that integrable three dimensional linearly degenerate quasilinear equations of a second order possess infinitely many differential constraints. Corresponding dispersive reductions are integrable systems associated with the Energy Dependent Schrodinger operator.

On classification and applications of pencils of Lie algebras

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In 1987 Petro Holod wrote a pioneering work where he investigated the Landau-Lifshitz equations using certain modification of the classical r-matrix approach. A key ingredient in his considerations are the so-called Lie pencils, i.e. linear families of Lie brackets on a given vector space. The talk will be devoted to an overview of applications of Lie pencils to integrable systems in frames of bihamiltonian approach and to recent results on classification of Lie pencils on semisimple Lie algebras.

Dynamics of Dicke model via generalized theta-functions

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We study semi-classical version of Dicke model which describes the system of N independent two-level atoms interacting with a single-mode electromagnetic field. It is known to be algebraically integrable, so one can use the ordinary scheme of separation of variables (SoV) in terms of which standart Jacobi inversion problem can be formulated and solved. But as the number of separated variables in this case is less than number of initial dynamical variables additional integration is needed in order to recover full dynamics. To avoid it we propose to use the modified scheme of SoV and introduce the proper number of separated variables. It implies solving the Jacobi inversion problem on a generalized Jacobian. In this case one has extended Abel-Jacobi map, namely, the Jacobi inversion problem includes not only holomorphic differentials but also some number (in our case, one) of Abelian differentials of the third kind. Following approach of Yu. Fedorov, we express the root functions (symmetric functions in separated variables and branching points of the algebraic curve) in terms of theta-quotient and certain exponentials. That can be done for the arbitrary genus on the basis of the concept of generalized theta-function.

Lie-Poisson geometry of flag manifolds defined by linear transformations of $\mathbb{C}^N$

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Any linear transformation of $\mathbb{C}^N$ with non-trivial Jordan structure define set of subspaces, many ordered subsets of this set form flags. I restrict the consideration on flags $F_0 \subset F_1 \subset \cdots \subset F_{m^A} \subset F_{m^A+1} = \mathbb{C}^N$ with two additional properties. A first one is: A can be contracted on any space F_k from the flag, and a second one is: a natural action of A on F_k/F_{k-1} is a transformation proportional to the unit transformation. I will construct a special coordinate-set of functions on such a flag. The functions define a Lagrangian manifold with respect to the Lie-Poisson structure on the orbit. The functions define submanifold of the orbit in a sense that the submanifold in question is a common constant-level set of the functions. I describe the construction of these functions and functions canonically conjugated to them. The set of coordinate functions is birational with respect to the matrix elements of A if the eigenvalues are given.

Spectral characteristics of Riemann surfaces with polyhedral metrics

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The boundary of a (not simply connected) polyhedron in $\mathbb{R}^3$ can be considered as a Riemann surface with conformal flat singular metric. We will discuss the spectral theory of the Laplace operator on such a surface. The talk is based on the joint works with D. Korotkin, L. Hillairet and V. Kalvin.

Extended KP hierarchies with self-consistent sources and binary Darboux transformations

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Extensions of the k-constrained KP hierarchy (k-cKPH) are presented. These hierarchies cover (2+1)-dimensional extensions of the k-cKPH and include matrix Davey-Stewartson equation, generalization of the N-wave system and extended KP equation with self-consistent sources. Binary Darboux transformations are applied to construct solutions of the presented hierarchies.

Recursion operator for Adler's equation in the Viallet form

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In this talk, we present two recursion operators for Adler's equation in the Viallet form. They satisfy the elliptic curve equation associated with the equation.
